Joint Relay Selection and Resource Allocation for Delay-Sensitive Traffic in Multi-Hop Relay Networks

Yan Sha[1*], Jufeng Hu[1], Shuang Hao[2], and Dan Wang[1*]
[1]School of Medical Information & Engineering, Xuzhou Medical University
Xuzhou 221004, China
[e-mail: uestcsy2009@126.com, hjf098@xzhmu.edu.cn, ndskywd@126.com]
[2]Chengdu University of TCM
Chengdu 610000, China;
[e-mail: 514302695@qq.com]
[*]Corresponding author: Yan Sha, Dan Wang

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Abstract

In this paper, we investigate traffic scheduling for a delay-sensitive multi-hop relay network, and aim to minimize the priority-based end-to-end delay of different data packet via joint relay selection, subcarrier assignment, and power allocation. We first derive the priority-based end-to-end delay based on queueing theory, and then propose a two-step method to decompose the original optimization problem into two sub-problems. For the joint subcarrier assignment and power control problem, we utilize an efficient particle swarm optimization method to solve it. For the relay selection problem, we prove its convexity and use the standard Lagrange method to deal with it. The joint relay selection, subcarriers assignment and transmission power allocation problem for each hop can also be solved by an exhaustive search over a finite set defined by the relay sensor set and available subcarrier set. Simulation results show that both the proposed routing scheme and the resource allocation scheme can reduce the average end-to-end delay.

Keywords: Multi-hop relay network, queue theory, relay selection, resource allocation

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1. Introduction

Relay-assisted OFDM-based networks are expected to provide ubiquitous high-data-rate coverage with efficient spectrum-utilization. Combining OFDM with a relay system can bring better design flexibility and enable multi-carrier systems to be deployed in a short time [1]. To achieve this objective, efficient resource allocation schemes are critical. Unlike traditional single-hop OFDM or OFDMA systems, the channel state of a subcarrier is location dependent and differs according to the relay sensors in relay-assisted OFDM-based networks. Thus, the coordination between resource allocation and relay selection is of fundamental importance. Though joint relay selection and resource allocation (JRSRA) algorithms for two-hop OFDM relaying networks have been studied extensively in recent years, the increased number of relaying hops brings us more advantages, but typically greater design challenges. For example, compared with two-hop OFDM relaying networks, multi-hop OFDM relaying networks can further extend the area of coverage and improve capacity. However, with the increasing of relaying hops, both the design complexities and the end-to-end transmission delay increase. Due to the randomness of energy reaching each relay, relay selection planning is essential to improve the energy efficiency of users [2]. Hence, this paper focuses on optimizing the long-term end-to-end delay for a multi-hop OFDM relaying network. We propose to solve the joint relay selection, subcarriers assignment and transmission power allocation problem in a decentralized manner, with consideration of relay fairness.

The restriction that the source sensors and the relay sensors transmit in two consecutive disjoint time slots is often imposed for two-hop OFDM relaying networks, giving rise to the so-called half-duplex relaying scheme [3, 4]. The works in studies joint resource allocation problem in multiuser multicarrier relay networks where all the nodes communicate with each other through a single half-duplex relay node [5].

Though such a scheme makes relaying options much more convenient to implement for a two-hop network, it may induce significant delay and give rise to the hidden stations problem in a multi-hop network. Compared with full-duplex technique, half-duplex technique shows its disadvantages in terms of spectrum efficiency and symbol rate [6]. In this paper, the restriction that each sensor adopts a full-duplex mode and transmits over pre-assigned subcarriers is applied. Sensors are not constrained to transmit in specific time slots. The proposed subcarriers assignment scheme ensures no collision or interference.

Prior research has focused on centralized solutions for JRSRA in two-hop OFDM relaying networks [7-9]. These results are not scalable by network size or the number of sensors due to the decentralized-information nature of multi-hop networks. Moreover, the majority of research on JRSRA in two-hop OFDM relaying networks optimizes the system capacity on a time-frame basis [10-12]. Authors consider joint congestion control and power allocation in distributed wireless ad hoc networks [13]. Authors further research traffic-oriented resource allocation for mmwave multi-hop backhaul net-works and propose an algorithm based on matching theory to reduce the computational complexity [14].

These solutions may be suitable for a highly dynamic network environment or applications which emphasize short-term performance. However, for a low dynamic network environment or applications, emphasis on long-term performance such as multimedia video streaming, JRSRA algorithms that focus on long-term performance are more attractive. In addition, with the expansion of network scale, the state information cannot be obtained in a timely manner by each sensor, and frequent private information exchanges are cost-prohibitive. Hence, JRSRA algorithms which consider long-term performance metrics (with minimal information exchanges) and do not need a central-controller are very suitable for a multi-hop OFDM relaying network. Authors and investigate the long-term performance of delay-sensitive multimedia applications in multi-hop networks in a semi-distributed manner [15,16]. The works in studies multi-hop implementation and resource allocation over multi-access channels [17]. Authors design a scheme for multiple underlay cellular networks and derive the outage probability and an effective throughput of D2D communications in approximate forms [18]. However, they only explored partial resource allocation and therefore these results cannot be applied in multi-hop OFDM relaying networks.

Relay fairness plays an important role in load balancing among relay sensors. Most of the existing research assesses relay fairness based on short-term performance metrics such as power consumption or the number of assigned subcarriers of each relay sensor during a time-frame [19,20]. Paper focus on the max-min energy efficiency. Even in some recent research on non-orthogonal multiple access, only short-term performance metrics are optimized [21,22]. However, in these papers relay fairness is assessed based on the long-term queuing load [23]. We attempt to distribute the traffic load evenly among relay sensors so that no relay sensors are overloaded.

The key contributions of this paper are summarized as follows:

1) Unlike previous work which considered short-term performance metrics, we formulate a cross-layer optimization problem for a multi-hop OFDM relaying network to minimize the long-term end-to-end delay. Moreover, we show that the optimization problem can be effectively solved in a distributed manner via two steps:

i) joint subcarrier assignment and transmission power allocation (STA) problem for a given relay selection;

ii) relay selection (RS) for a given subcarrier and transmission power assignment.

2) The STA problem can be efficiently solved in two phases according to the total transmission power of each sensor. In particular, when the transmission power is sufficiently low, the long-term end-to-end delay is minimized when each sensor allocates its total transmission power to the best subcarrier, and the STA problem can be equivalently transformed into a linear optimization problem from the graphical perspective.

3) The RS problem is optimally and efficiently solved by using the Lagrange method, and the proposed iterative searching method not only gives a closed form of the optimal solution, but also provides a deep insight into the issues involved.

The remainder of this paper is organized as follows. In Section 2, we introduce the system model, and formulate a distributed optimization problem. The associated algorithms for the joint subcarriers, transmission power allocation and relay selection are introduced in Section 3 and Section 4, respectively. In Section 5, we present the algorithm for the joint subcarriers assignment, transmission power allocation and relay selection. Section 6 provides some simulation results to illustrate the performance of the proposed algorithms. Finally, we conclude this paper in Section 7.

2. System Model and Problem Formulation

2.1 System Model

We consider a H-th hop relaying network as shown in **Fig. 1**, where data packets are generated by the source sensor in the first hop and relayed hop-by-hop until the destination sensor in the H-th hop receives them. Each H-th hop consists of N_h sensors, for $h \in \{1,\ldots,H\}$. Each sensor in the network operates in full-duplex mode. The transmission occurs between the adjacent hops and no retransmissions are considered for lost packets. Since transmissions are based on

OFDM modulation, the spectrum resources are logically divided into multiple orthogonal subcarriers with slow fading. The available subcarriers for each H-th hop are assumed to be predetermined and number K_h, for $h \in \{1,...,H\}$. A statistical routing mechanism is used, in which sensors are randomly selected according to certain probabilities [15]. We assume that the input traffic at the source sensor follows a Poisson process. Thus, the arrival of traffic at each relay sensor can also be approximated by a Poisson process and the queuing model of each sensor is M/G/1 [15].

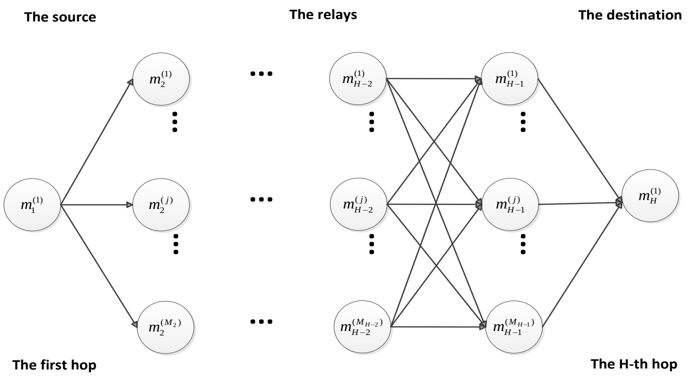

Fig. 1. Multi-hop multi-relay assisted OFDM system.

Let the sensor set and the available subcarrier set for the H-th hop be $\mathbf{N_h} = \{n_{h,i}\}_{1\le i\le N_h}$ and $\mathbf{F}_h = \{f_{h,k}\}_{1\le k\le K_h}$, for $h \in \{1,...,H\}$, respectively, where $n_{h,i}$ denotes the *i*-th sensor and $f_{h,k}$ denotes the *k*-th subcarrier. Define the subcarrier selection parameter of sensor $n_{h,i}$ as $C_{h,i,k} = \{0,1\}$, where $C_{h,i,k} = 1$ if subcarrier $f_{h,k}$ is assigned to $n_{h,i}$ and $C_{h,i,k} = 0$ otherwise. In order to avoid interference or collisions, we require the available subcarrier sets in coherent three hops are not overlapped, i.e., $\mathbf{F}_h \cap \mathbf{F}_{h+1} \cap \mathbf{F}_{h+2} = \varnothing$, and a subcarrier can not be assigned to more than one sensors in the same hop, i.e., $\sum_{i=1}^{N_h} C_{h,i,k} \le 1$.

We define the efficient channel gain of the link connecting $n_{h,i}$ to $n_{h+1,j}$ over subcarrier $f_{h,k}$ as $g_{h,i,j,k}$ [9]. This is a random variable following a distribution $\rho_{h,i,j,k}(x)$ so that the expectation of $g_{h,i,j,k}$ can be calculated via:

$$E[g_{h,i,j,k}] = \int_{g_{\min}}^{g_{\max}} x\rho_{h,i,j,k}(x)dx \tag{1}$$

Where $g_{\max}$ and $g_{\min}$ are boundaries of the observable value for $g_{i,j,k}$. To make further analysis easier, we require that $\rho_{h,i,j,k}(x) = \rho_{h,i,k}(x)$, for $j \in \{1,...,N_{h+1}\}$. This condition essentially implies that links which connect one sensor to its neighbors over a subcarrier hold the same magnitude of $E[g_{h,i,j,k}]$:

$$E[g_{h,i,j,k}] = E[g_{h,i,k}], \text{ for } j \in \{1,...,N_{h+1}\} \tag{2}$$

Let the transmission power allocated on $f_{h,j}$ by $n_{h,i}$ be $P_{h,i,k}$. The signal-to-noise-ratio (SNR) of link $(n_{h,i}, n_{h+1,j}, f_{h,k})$ can then be calculated via $u_{h,i,j,k} = P_{h,i,k} g_{h,i,j,k}$. The effective goodput in a link for the received $u_{h,i,j,k}$ is defined as $T(u_{h,i,j,k})$. The expression of $T(u_{h,i,j,k})$

depends on the modulation and coding schemes (MCS) and different MCS lead to different $T(u_{h,i,j,k})$. For some popular MCS, $T(u_{h,i,j,k})$ can be approximated as a sigmoid function of $u_{h,i,j,k}$ [24-26].

We define $\beta_{h+1,j}$ as the percentage of sensors in the H-th hop that select sensor $n_{h+1,j}$ as their relay. It is further defined that the average queuing delay of $n_{h,i}$ is $E[D_{h,i}]$, which can then be calculated via [23]:

$$E[D_{h,i}] = \begin{cases} \dfrac{\beta_{h,i} R E[S_{h,i}^2]}{2(1-\beta_{h,i} R E[S_{h,i}])} + E[S_{h,i}] & \text{if } \beta_{h,i} > 0 \\ 0 & \text{if } \beta_{h,i} = 0 \end{cases} \tag{3}$$

Where R is the arrival rate of the input traffic at the source. $S_{h,i}$ follows exponential distribution, and $E[S_{h,i}]$ and $E[S_{h,i}^2]$ are the first and second moments of the service time [27], respectively. Since different subcarriers are mutually independent, $E[S_{h,i}]$ can be expressed (assuming that the packet length is normalized) as:

$$E[S_{h,i}] = \sum_{j=1}^{N_{h+1}} \beta_{h+1,j} \left\{ \sum_{k=1}^{K_h} C_{h,i,k} E\left[T(u_{h,i,j,k})\right] \right\}^{-1} \tag{4}$$

Where $E\left[T(u_{h,i,j,k})\right]$ is the average effective goodput. Since no retransmissions are allowed, $E[S_{h,i}^2]$ can be calculated via:

$$E[S_{h,i}^2] = \sum_{j=1}^{N_{h+1}} \beta_{h+1,j} \left\{ \sum_{k=1}^{K_h} C_{h,i,k} E\left[T(u_{h,i,j,k})\right] \right\}^{-2} \tag{5}$$

2.2 A Distributed Problem Formulation

Based on the system model introduced in the last sub-section, we formulate the optimization problem as:

Problem 1:

$$\min_{\{\mathbf{B},\mathbf{C},\mathbf{P}\}} \sum_{h=1}^{H-1} \sum_{i=1}^{N_h} \beta_{h,i} E[D_{h,i}] \tag{6}$$

Subject to:

$$\sum_{i=1}^{N_h} \beta_{h,i} = 1, \forall h \tag{7}$$

$$\sum_{k=1}^{K_h} P_{h,i,k} \le P_{h,i}, \forall h,i \tag{8}$$

$$\sum_{i=1}^{N_h} C_{h,i,k} \le 1, \forall h,k \tag{9}$$

$$1-\beta_{h,i} R E[S_{h,i}] > 0, \forall n_{h,i} \tag{10}$$

The second constraint implies that the total transmission power of each sensor is limited and the fourth constraint is to guarantee that the average queuing delay is non-negative [23].

Due to the decentralized-information nature of multi-hop networks, a centralized solution may not be practical for Problem 1. Therefore, it is necessary to establish a distributed solution. To fulfill this goal, the following approximation is adopted:

$$E\left[T(u_{h,i,j,k})\right] = T\left(\bar{u}_{h,i,k}\right) \tag{11}$$

Where $\bar{u}_{h,i,k} = E[g_{h,i,k}]P_{h,i,k}$. This approximation is essentially based on the fact that each subcarrier is slow fading. The invariance of $E[g_{h,i,k}]$ ensures $T\left(\bar{u}_{h,i,k}\right)$ is a sigmoid function in $P_{h,i,k}$.

Substituting (11) into (3), the corresponding $E[D_{h,i}]$ can be expressed as:

$$E[D_{h,i}] = \begin{cases} \dfrac{\beta_{h,i} R \left\{ \sum_{k=1}^{K_h} C_{h,i,k} T\left(\bar{u}_{h,i,k}\right) \right\}^{-2}}{2\left\{ 1 - \beta_{h,i} R \left\{ \sum_{k=1}^{K_h} C_{h,i,k} T\left(\bar{u}_{h,i,k}\right) \right\}^{-1} \right\}} + \left\{ \sum_{k=1}^{K_h} C_{h,i,k} T\left(\bar{u}_{h,i,k}\right) \right\}^{-1} & if\ \beta_{h,i} > 0 \\ 0 & if\ \beta_{h,i} = 0 \end{cases} \tag{12}$$

Obviously, $E[D_{h,i}]$ is dominated by the cross-layer transmission options of sensors in the H-th hop. Therefore, Problem 1 can be decomposed into:
Problem 2

$$\min_{\{\mathbf{B}_h, \mathbf{C}_h, \mathbf{P}_h\}} \mathbf{U}_h(\mathbf{B}_h, \mathbf{C}_h, \mathbf{P}_h) \tag{13}$$

Subject to:

$$\sum_{i=1}^{N_h} \beta_{h,i} = 1 \tag{14}$$

$$\sum_{k=1}^{K_h} P_{h,i,k} \le P_{h,i}, \forall i \tag{15}$$

$$\sum_{i=1}^{N_h} C_{h,i,k} \le 1, \forall k \tag{16}$$

$$T\left(\bar{u}_{h,i,k}\right) > \beta_{h,i} R, \forall n_{h,i} \tag{17}$$

Where $\{\mathbf{B}_h, \mathbf{C}_h, \mathbf{P}_h\}$ is the cross-layer transmission options of sensors in the H-th hop, and $\mathbf{U}_h(\mathbf{B}_h, \mathbf{C}_h, \mathbf{P}_h) = \sum_{i=1}^{N_h} \beta_{h,i} E[D_{h,i}]$.

Motivated by the analysis in [28], we propose to solve Problem 2 by decomposing it into two sub-problems: one for tuning the assignment of subcarriers and transmission power for a given relay selection and the other for tuning the relay selection for a fixed assignment of joint subcarriers and transmission power. In the following sections, we will introduce the solutions for the two sub-problems, respectively.

3. Optimization by Resource Allocation

3.1 The Optimization Problem Formulation

Under given transmission schemes in the network layer, the STA problem is formulated as:
Problem 3

$$\min_{\{\mathbf{C}_h, \mathbf{P}_h\}} \sum_{i=1}^{N_h} \beta_{h,i} E[D_{h,i}] \tag{18}$$

Subject to:

$$\sum_{k=1}^{K_h} T\left(\bar{u}_{h,i,k}\right) > \beta_{h,i} R, \forall n_{h,i} \in \mathbf{N_h} \tag{19}$$

$$\sum_{k=1}^{K_h} P_{h,i,k} \le P_{h,i}, \forall i \tag{20}$$

$$\sum_{i=1}^{N_h} C_{h,i,k} \le 1, \forall k \tag{21}$$

Obviously, Problem 3 is combinational and nonlinear, and the computational complexity is very high when N_h and K_h are large. Hence, to make the analysis more tractable, we will firstly investigate the transmission power allocation algorithm for assigned subcarriers.

3.2 Transmission Power Allocation for Assigned Subcarriers

Let F' be the assigned subcarrier set for $n_{h,i}$. Since $E[D_{h,i}]$ decreases with an increase in $T\left(\bar{u}_{h,i,k}\right)$ over any subcarriers, minimizing $E[D_{h,i}]$ is equivalent to maximizing $\sum_{k=1}^{K_h} T\left(\bar{u}_{h,i,k}\right)$.

Assuming the feasible set satisfying constraint (19) always exists, the transmission power allocation problem for assigned subcarriers can be formulated as:

Problem 4

$$\max_{\{\mathbf{P}\}} \sum_{k\in F'} T\left(\bar{u}_{h,i,k}\right) \tag{22}$$

Subject to:

$$\sum_{k\in F'} P_{h,i,k} = P_{h,i} \tag{23}$$

Let us define $\lambda_k^{\max}$ as:

$$\lambda_k^{\max} = \min\left\{\lambda \ge 0 \left| \max_{0\le P\le P_{h,i}} \left\{T\left(E[g_{h,i,k}]P\right) - \lambda P\right\} = 0\right.\right\}, \text{for } k \in F' \tag{24}$$

We can calculate $\lambda_k^{\max}$ by solving the following equations :

$$\lambda_k^{\max} = \begin{cases} \dfrac{dT\left(E[g_{h,i,k}]P\right)}{dP}\Big|_{P=P'}, \\ if\ 0 < P'_{h,i,k} < P_{h,i} and\ P' exists \\ \dfrac{T\left(E[g_{h,i,k}]P_{h,i}\right)}{P_{h,i}}, \\ if\ P'_{h,i,k} = P_{h,i} or\ no\ P' exists \end{cases} \tag{25}$$

Where $P'_{h,i,k}$ is the inflection point of $T\left(E[g_{h,i,k}]P\right)$, and P' is the solution for the following equation:

$$T\left(E[g_{h,i,k}]P\right) - P\frac{dT\left(E[g_{h,i,k}]P\right)}{dP} = 0, P'_{h,i,k} < P < P_{h,i} \tag{26}$$

Based on the obtained $\lambda_k^{\max}$ from (25), Problem 4 can be efficiently solved by using the MSA algorithm and the power allocation algorithm . In particular, if

$$\frac{dT\left(E[g_{h,i,j}]P\right)}{dP}\Big|_{P=P_{h,i}} \ge \lambda_j^{\max} \text{ for } j = \arg\max_{k\in F'}(E[g_{h,i,k}]) \tag{27}$$

Then the optimal power allocation for Problem 4 is set as $P_{h,i,j} = P_{h,i}$.

Before proceeding to actual optimization, we present the following propositions about the optimal solution for Problem 4.

Proposition 1: Let $P'_{h,i,k}$ be the x-axis value of the turning point for the function. If the total transmission power $P_{h,i}$ satisfies $P_{h,i} \in \left[0, \min_{k\in F'}(P'_{h,i,k})\right]$, the optimal solution for Problem 4 is:

$$P^*_{h,i,k} = \begin{cases} P_{h,i} & if: k = \arg\max_{k\in F'}\left\{E[g_{h,i,k}]\right\} \\ 0 & else \end{cases} \tag{28}$$

Proof: The proof is in Appendix A.

Based on Proposition 1, Problem 4 can be solved in two phases according to the value of $P_{h,i}$ .Firstly, for $P_{h,i} \in \left[0, \min_{k\in F'}(P'_{h,i,k})\right]$, the optimal solution for Problem 4 is to allocate $P_{h,i}$ to the subcarrier, if and only if it holds the maximal $E[g_{h,i,k}]$ based on (28). Secondly, for $P_{h,i} \in \left(\min_{k\in F'}(P'_{h,i,k}), \infty\right)$, $T\left(\bar{u}_{h,i,k}\right)$ is neither concave nor convex and it turns out to be a nonlinear optimization problem, which can be solved by nonlinear programming methods such as a Particle Swarm Optimization (PSO) algorithm [29].

3.3 Joint Subcarrier and Transmission Power Allocation

Motivated by the analysis in the last sub-section, the optimization for Problem 3 can also be done in two phases.

1) $P_{h,i} \in \left(0, \min_{k}(P'_{h,i,k})\right]$, for $\forall n_{h,i} \in \mathbf{n_h}$

In this region, the optimal transmission power allocation algorithm for assigned subcarriers is to allocate its total transmission power to the best subcarrier. However, a subcarrier may be preferred by multiple sensors, while only one subcarrier-sensor coalition is allowed. Hence, the STA problem can be transformed into a linear optimization problem to decide the optimal subcarrier-sensor coalitions.

Let $\mathbf{B}_h$ be the loss matrix of each H-th hop with element b_{ik} representing the delay of $n_{h,i}$ when subcarrier $f_{h,k}$ is assigned to it and the transmit power is also given. The loss for assignments which violate constraint (19) are assumed to be infinite. Thus, b_{ik} can be calculated via:

$$b_{ik} = \begin{cases} \dfrac{\beta^2_{h,i} R\left\{T\left(\bar{u}_{h,i,k}\right)\right\}^{-2}}{2 - 2\beta_{h,i} R\left\{T\left(\bar{u}_{h,i,k}\right)\right\}^{-1}} + \beta_{h,i}\left\{T\left(\bar{u}_{h,i,k}\right)\right\}^{-1} & if: T\left(\bar{u}_{h,i,k}\right) > \beta_{h,i} R \\ \infty & else \end{cases} \tag{29}$$

Hence, the assignment problem is how to select the sensor-subcarrier pairs so that the overall benefit is minimized, which is stated as:

Problem 5

$$\min_{\{\mathbf{C}\}} \sum_{k=1}^{K_h} \sum_{i=1}^{N_h} C_{h,i,k} b_{ik} \tag{30}$$

Subject to:

$$\sum_{i=1}^{N_h} C_{h,i,k} \le 1, \forall k \tag{31}$$

$$\sum_{k=1}^{K_h} C_{h,i,k} = 1, \forall i \tag{32}$$

One of the possible solutions for Problem 5 is the Hungarian method [30].

Based on the analysis above, we present the joint subcarrier and transmission power allocation algorithm for $\forall P_{h,i} \in \left(0, \min_k (P'_{h,i,k})\right]$ as follows:

Algorithm 2: the joint subcarrier and transmission power allocation for $\forall P_{h,i} \in \left(0, \min_k (P'_{h,i,k})\right]$

Step 1: Build the loss matrix $\mathbf{B}_h$ for the H-th hop according to (29).

Step 2: Calculate the optimal coalition $\mathbf{C_h}$ for $\mathbf{B}_h$ by using the Hungarian method.

Step 3: Allocate the transmission power as $P_{h,i,k} = \begin{cases} P_{h,i} & if: C_{h,i,k} = 1 \\ 0 & if: C_{h,i,k} = 0 \end{cases}$

2) $P_{h,i} \in \left(\min_k (P'_{h,i,k}), \infty\right)$, for $\exists n_{h,i} \in \mathbf{n_h}$

In this region, we propose a heuristic algorithm to solve the STA problem. Firstly, we do an initial assignment of subcarriers and transmission power for each sensor by using the approach mentioned in (1). After this process, constraint (19) is satisfied. Then, we assign the unallocated subcarriers one subcarrier at a time to those sensors with $P_{h,i} \in \left(\min_k (P'_{h,i,k}), \infty\right)$. In each assignment, the transmission power is allocated by using a PSO method and the sensor-subcarrier pair which leads to the maximal decrease of the average queuing delay is selected. The joint subcarriers assignment process stops when all K_h subcarriers are assigned.

Based on introductions above, the joint subcarrier assignment and transmission power allocation algorithm can be summarized as the following:

Algorithm 3: the joint subcarrier and transmission power allocation algorithm for $\exists P_{h,i} \in \left(\min_k (P'_{h,i,k}), \infty\right)$

Step 1: Assign subcarriers and the transmission power by using Algorithm 2 and calculate $\Delta_{m_h^{(i)}} = \beta_{m_h^{(i)}} E[D_{m_h^{(i)}}]$, for $\forall m_h^{(i)}$.

Step 2: For any unallocated subcarrier $f_h^{(k)}$, each sensor $m_h^{(i)}$ calculates $\Delta'_{m_h^{(i)}} = \beta_{m_h^{(i)}} E[D_{m_h^{(i)}}]$, assuming $f_h^{(k)}$ is allocated to $m_h^{(i)}$.

Step 3: Let $C_{m_h^{(j)}, f_h^{(k)}} = 1$, if $j = \arg\max_i \{\Delta'_{m_h^{(i)}} - \Delta_{m_h^{(i)}}\}$, and update the transmission power allocation matrix $\mathbf{P_h}$.

Step 4: Repeat Steps 2 and 3, until all the subcarriers are allocated.

4. Optimization for Relay Selection

4.1 Optimization Problem Formulation

Under given transmission schemes in the physical layer, the optimization problem for the network layer is formulated as the following:

Problem 6

$$\min_{\mathbf{B_h}} \sum_{i=1}^{M_h} \left\{ \frac{\beta^2_{m_h^{(i)}} R \overline{T}^{-2}_{m_h^{(i)}}}{2[1-\beta_{m_h^{(i)}} R \overline{T}^{-1}_{m_h^{(i)}}]} + \beta_{m_h^{(i)}} \overline{T}^{-1}_{m_h^{(i)}} \right\} \tag{33}$$

Subject to:

$$\sum_{i=1}^{M_h} \beta_{m_h^{(i)}} = 1 \tag{34}$$

$$0 \le \beta_{m_h^{(i)}} \le 1, \forall m_h^{(i)} \in \mathbf{m_h} \tag{35}$$

$$\beta_{m_h^{(i)}} R \overline{T}_{m_h^{(i)}}^{-1} - 1 < 0, \forall m_h^{(i)} \in \mathbf{m_h} \tag{36}$$

Where constraint (36) is to guarantee that no sensor is overloaded, and $\overline{T}_{m_h^{(i)}}$ denotes the average effective transmission rate of $m_h^{(i)}$. As the transmission schemes in the physical layer are given, $\overline{T}_{m_h^{(i)}}$ is invariant, and can be calculated via:

$$\overline{T}_{m_h^{(i)}} = \sum_{f_h^{(k)} \in \mathbf{F_h}} T\left(E[g_{m_h^{(i)}, f_h^{(k)}}] P_{m_h^{(i)}, f_h^{(k)}}\right) \tag{37}$$

For simplification, in this section we sort sensors in the H-th hop based on the value of $\overline{T}_{m_h^{(i)}}$, and arrange indexes from largest to smallest, i.e.

$$\overline{T}_{m_h^{(i)}} \ge \overline{T}_{m_h^{(j)}}, \text{ for } i < j \tag{38}$$

To make the further analysis easier, we present the following proposition about the feasible solution space of Problem 6, respectively.

Proposition 2: The feasible solution space for Problem 6 is not empty, if and only if $\sum_{i=1}^{M_h} \overline{T}_{m_h^{(i)}} > R$.

Proof: The proof is given in Appendix B.

4.2 Optimization by Using The Lagrange Method

Since Problem 6 is obviously a convex optimization problem, it can be solved by a sub-gradient method [31]. However, this method requires a huge number of iterations, which is cost inefficient, and moreover it provides no insights into the correlations between resource allocation and relay selection. Thus, it is necessary to establish a method which not only can efficiently solve Problem 6, but also gives a better understanding of the issues involved.

Applying the Lagrange method, we obtain:

$$L\left(\beta_{m_h^{(i)}}, \partial\right) = \sum_{i=1}^{M_h} \frac{2\beta_{m_h^{(i)}} \overline{T}_{m_h^{(i)}} - R\beta_{m_h^{(i)}}^2}{2\overline{T}_{m_h^{(i)}}^2 - 2R\beta_{m_h^{(i)}} \overline{T}_{m_h^{(i)}}} - \partial\left(\sum_{i=1}^{M_h} \beta_{m_h^{(i)}} - 1\right) \tag{39}$$

Where ∂ is the Lagrange multiplier for constraint (34).

After differentiating L with respect to $\beta_{m_h^{(i)}}$, we obtain the necessary condition for the optimal solution $\beta_{m_h^{(i)}}^*$:

$$\begin{aligned} &\frac{\partial L}{\partial \beta_{m_h^{(i)}}} \Big|_{\beta_{m_h^{(i)}} = \beta_{m_h^{(i)}}^*} \\ &= \frac{R^2 \beta_{m_h^{(i)}}^2 - 2R\beta_{m_h^{(i)}} \overline{T}_{m_h^{(i)}} + 2\overline{T}_{m_h^{(i)}}^2}{2(R^2 \overline{T}_{m_h^{(i)}} \beta_{m_h^{(i)}}^2 - 2R\beta_{m_h^{(i)}} \overline{T}_{m_h^{(i)}}^2 + \overline{T}_{m_h^{(i)}}^3)} - \partial^* \begin{cases} \ge 0 & if\ \beta_{m_h^{(i)}}^* = 0 \\ = 0 & if\ \beta_{m_h^{(i)}}^* \in (0,1] \end{cases} \end{aligned} \tag{40}$$

Where ∂^* is the corresponding Lagrange multiplier for $\beta_{m_h^{(i)}}^*$. The necessary condition can be interpreted by the fact that if the minimum happens to occur in the constraint region $(0,1]$, then the derivative evaluated at the minimum point must be equal to zero. On the other hand, if the minimum occurs at a boundary point of $\beta_{m_h^{(i)}}^* = 0$, for $\exists m_h^{(i)} \in \mathbf{m_h}$, then the derivative must be equal to zero or positive along all directions pointing towards the interior of the constraint set.

Some helpful propositions for the optimal solution are presented as follows:

Proposition 3: Supposing $\beta^*_{m_h^{(i)}} > 0$, for $\forall m_h^{(i)} \in \mathbf{m_h}$, then the sufficient and necessary condition for $\beta^*_{m_h^{(i)}}$ is $\frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}}|_{\beta_{m_h^{(i)}}=\beta^*_{m_h^{(i)}}} = \partial^*$, for $\forall m_h^{(i)} \in \mathbf{m_h}$, where $\partial^* > \overline{T}^{-1}_{m_h^{(M_h)}}$ and $\Lambda = \sum_{i=1}^{M_h} \left\{ \frac{\beta^2_{m_h^{(i)}} R \overline{T}^{-2}_{m_h^{(i)}}}{2[1-\beta_{m_h^{(i)}} R \overline{T}^{-1}_{m_h^{(i)}}]} + \beta_{m_h^{(i)}} \overline{T}^{-1}_{m_h^{(i)}} \right\}$.

Proof: The proof is given in Appendix C.

Proposition 4: The optimal solution $\beta^*_{m_h^{(i)}}$ should satisfy $\beta^*_{m_h^{(i)}} \geq \beta^*_{m_h^{(j)}}$, for $\forall j > i$.

Proof: The proof is given in Appendix D.

Proposition 5: Supposing there is $n \leq M_h$ so that $\begin{cases} \beta^*_{m_h^{(i)}} > 0 & for\, i \leq n \\ \beta^*_{m_h^{(i)}} = 0 & for\, i > n \end{cases}$, then the sufficient and necessary condition for $\beta^*_{m_h^{(i)}}$ is $\begin{cases} \overline{T}^{-1}_{m_h^{(n)}} < \frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}}|_{\beta_{m_h^{(i)}}=\beta^*_{m_h^{(i)}}} \leq \overline{T}^{-1}_{m_h^{(n+1)}} & for\, i \leq n \\ \frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}}|_{\beta_{m_h^{(i)}}=\beta^*_{m_h^{(i)}}} = \frac{\partial \Lambda}{\partial \beta_{m_h^{(j)}}}|_{\beta_{m_h^{(j)}}=\beta^*_{m_h^{(j)}}} & for\, i, j \leq n \end{cases}$, where $\Lambda = \sum_{i=1}^{M_h} \left\{ \frac{\beta^2_{m_h^{(i)}} R \overline{T}^{-2}_{m_h^{(i)}}}{2[1-\beta_{m_h^{(i)}} R \overline{T}^{-1}_{m_h^{(i)}}]} + \beta_{m_h^{(i)}} \overline{T}^{-1}_{m_h^{(i)}} \right\}$.

Proof: The proof is given in Appendix E

Let $\mathbf{m_h^*}$ be the set of sensors whose optimal solution is greater than zero. According to Proposition 5, we can conclude that:

$$\partial^* \begin{cases} > \overline{T}^{-1}_{m_h^{(i)}} & if\ m_h^{(i)} \in \mathbf{m_h^*} \\ \leq \overline{T}^{-1}_{m_h^{(i)}} & if\ m_h^{(i)} \notin \mathbf{m_h^*} \end{cases} \tag{41}$$

Solving (40), the optimal solution $\beta^*_{m_h^{(i)}}$ can be expressed as:

$$\beta^*_{m_h^{(i)}} = \begin{cases} \frac{\overline{T}_{m_h^{(i)}}}{R}\left[1-(2\partial^* \overline{T}_{m_h^{(i)}} - 1)^{-0.5}\right] & if\ \partial^* > \overline{T}^{-1}_{m_h^{(i)}} \\ 0 & if\ \partial^* \leq \overline{T}^{-1}_{m_h^{(i)}} \end{cases} \tag{42}$$

Or

$$\beta^*_{m_h^{(i)}} = \begin{cases} \frac{\overline{T}_{m_h^{(i)}}}{R}\left[1+(2\partial^* \overline{T}_{m_h^{(i)}} - 1)^{-0.5}\right] & if\ \partial^* > \overline{T}^{-1}_{m_h^{(i)}} \\ 0 & if\ \partial^* \leq \overline{T}^{-1}_{m_h^{(i)}} \end{cases} \tag{43}$$

Since constraint (36) implies that $\beta_{m_h^{(i)}} < \frac{\overline{T}_{m_h^{(i)}}}{R}$, expression (43) is infeasible. Substituting (42) into (34), we obtain:

$$\sum_{m_h^{(i)} \in \mathbf{m_h^*}} \frac{\overline{T}_{m_h^{(i)}}}{R}\left[1-(2\partial^* \overline{T}_{m_h^{(i)}} - 1)^{-0.5}\right] = 1 \tag{44}$$

4.3 The Iterative Search Algorithm

Obviously, the relay selection problem is equivalently transformed into establishing the optimal ∂^* or $\mathbf{m}_\mathbf{h}^*$. We propose an iterative search algorithm to solve it. The iterative procedure starts by setting $\mathbf{m}_\mathbf{h}^* = \mathbf{m}_\mathbf{h}$, and then establishes the corresponding ∂^* by solving (44). If there are $\overline{T}_{m_h^{(i)}} < \frac{1}{\partial^*}$, the algorithm excludes those sensors from $\mathbf{m}_\mathbf{h}^*$, and then solves equation (44) with the updated $\mathbf{m}_\mathbf{h}^*$. The iterative process repeats until $\overline{T}_{m_h^{(i)}}$, for $\forall m_h^{(i)} \in \mathbf{m}_\mathbf{h}^*$, is greater than $\frac{1}{\partial^*}$, and then we substitute ∂^* into (42) to obtain the optimal solution $\beta^*_{m_h^{(i)}}$. The iterative search algorithm is summarized as the following:

Algorithm 4: The iterative search algorithm

Step 1: Set $\mathbf{m}_\mathbf{h}^* = \mathbf{m}_\mathbf{h}$.

Step 2: Solve equation (44) to obtain ∂^*.

Step 3: If ∂^* does not satisfy (41), define the sensor set $\mathbf{m}'_\mathbf{h}$ for those sensors with $\overline{T}_{m_h^{(i)}} > \frac{1}{\partial^*}$, and update $\mathbf{m}_\mathbf{h}^*$ as $\mathbf{m}_\mathbf{h}^* = \mathbf{m}_\mathbf{h}^* - \mathbf{m}'_\mathbf{h}$. On the other hand, if ∂^* satisfies (41), then skip to Step 5.

Step 4: Repeat Steps 2 and 3 until the obtained ∂^* satisfies (41).

Step 5: Substitute ∂^* into (42) to calculate $\beta^*_{m_h^{(i)}}$, for $\forall m_h^{(i)} \in \mathbf{m}_\mathbf{h}$.

Let $\mathrm{G}(\partial^*) = \sum_{m_h^{(i)} \in \mathbf{m}_\mathbf{h}^*} \frac{\overline{T}_{m_h^{(i)}}}{R}\left[1-(2\partial^*\overline{T}_{m_h^{(i)}}-1)^{-0.5}\right]$. The monotonicity of $\mathrm{G}(\partial^*)$ and the observation that $\mathrm{G}(\partial^*)$ is continuous with $\mathrm{G}(0) = \sum_{m_h^{(i)} \in \mathbf{m}_\mathbf{h}^*} \frac{\overline{T}_{m_h^{(i)}}}{R}$ and $\mathrm{G}(\infty) = 0$, ensure that a unique solution ∂^* can be obtained by using a root-finding method, such as Newton's method [32], in each iteration. If Proposition 2 is satisfied, the optimal ∂^* always exists, and the value of the obtained ∂^* decreases with the increase in the number of iterations. Hence, Algorithm 4 is convergent, and the computational complexity is $M_h - 1$ at most.

5. Joint Relay Selection and Resource Allocation

This section is not mandatory but can be added to the manuscript if the discussion is unusually long or complex. The optimal transmission power allocation for given subcarrier assignments can be obtained by using Algorithm 1, while the optimal relay selection for predetermined transmission power and subcarrier assignments can be obtained via Algorithm 4. Consequently, the joint relay selection, subcarriers assignment and transmission power allocation problem for the H-th hop can be solved by an exhaustive search over a finite set defined by relay sensor set $\mathbf{m}_\mathbf{h}$ and available subcarrier set $\mathbf{F}_h$. The size of the search set is a product of the number of sensors and available subcarriers. When the number of sensors and subcarriers are large, the complexity of the exhaustive search is very high. Thus, we propose a lower complexity iterative method to approach the optimal solution, and it is summarized as follows:

Algorithm 5: Joint relay selection and resource allocation algorithm

Step 1: Set $n=1$, and $\beta_{m_h^{(i)}}(n)=\frac{1}{M_h}$, for $m_h^{(i)} \in \mathbf{m_h}$. Calculate $\{\mathbf{C}_h(n), \mathbf{P}_h(n)\}$ using Algorithm 2 based on $\mathbf{B_h}(n)$

Step 2: Set $n=n+1$. Calculate $\mathbf{B_h}(n)$ using Algorithm 4 based on $\{\mathbf{C}_h(n-1), \mathbf{P}_h(n-1)\}$, and if $\mathbf{B_h}(n)=\mathbf{B_h}(n-1)$, skip to Step 4.

Step 3: If there are $P_{m_h^{(i)}} \in \left(0, \min_{f_h^{(k)}}(P'_{m_h^{(i)}, f_h^{(k)}})\right]$, for $\forall m_h^{(i)} \in \mathbf{m_h}$, then calculate $\{\mathbf{C}_h(n), \mathbf{P}_h(n)\}$ using Algorithm 2 based on $\mathbf{B_h}(n)$. On the other hand, if there is $P_{m_h^{(i)}} \in \left(\min_{f_h^{(k)}}(P'_{m_h^{(i)}, f_h^{(k)}}), \infty\right)$, for $m_h^{(i)} \in \mathbf{m_h}$, then calculate $\{\mathbf{C}_h(n), \mathbf{P}_h(n)\}$ using Algorithm 3 based on $\mathbf{B_h}(n)$. If $\{\mathbf{C}_h(n), \mathbf{P}_h(n)\} \neq \{\mathbf{C}_h(n-1), \mathbf{P}_h(n-1)\}$, then skip back to Step 2.

Step 4: The iteration stops, and the optimal strategy is $\{\mathbf{B_h}(n), \mathbf{C}_h(n), \mathbf{P}_h(n)\}$.

In each iteration, the utility function $\mathbf{U}_h$ is non-increasing in Steps 2 and 3, and it is lower bounded. Hence, Algorithm 6 is also convergent.

6. Simulation Results

This section presents the performance evaluation of the proposed transmission scheme summarized in Section V, which minimizes the average end-to-end delay via joint relay selection, subcarriers assignment and transmission power allocation. We consider a four-hop relay-assisted OFDM network as shown in **Fig. 2**. In the network, the modulation scheme is 64-QAM, and the coding rate is 2/3. The average effective goodput can be calculated via [24]:

$$T\left(E[g_{m_h^{(i)}, f_h^{(k)}}]P_{m_h^{(i)}, f_h^{(k)}}\right) = 48\left\{1+\exp\left[-0.625(10\lg(E[g_{m_h^{(i)}, f_h^{(k)}}]P_{m_h^{(i)}, f_h^{(k)}})-18.2\right]\right\}^{-1} \tag{45}$$

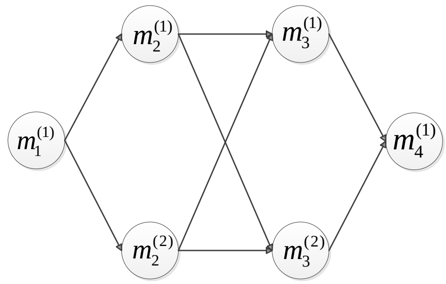

Fig. 2. The four-hop relay assisted network

The expectation of the efficient channel gains between two adjacent hops is defined as

$$H_h = \left(E[g_{m_h^{(i)}, f_h^{(k)}}]\right)_{1\le i\le M_h, 1\le k\le K_h}, \text{ for } h=(1,2,3) \tag{46}$$

With

$$H_1 = \begin{pmatrix} 0.9 \\ 0.7 \end{pmatrix} \tag{47}$$

$$H_2 = \begin{pmatrix} 0.95 & 0.8 & 0.8 \\ 0.7 & 0.9 & 0.7 \end{pmatrix} \tag{48}$$

$$H_3 = \begin{pmatrix} 0.65 & 0.55 & 0.55 \\ 0.55 & 0.6 & 0.55 \end{pmatrix} \tag{49}$$

In addition, all the relays and the source are assumed to be subject to the same transmission power constraint for simplicity, and the total transmission power of each sensor ranges from 50 w to 200w.

For the purpose of performance comparison, we consider the following transmission schemes:

Scheme 1: the proposed transmission scheme summarized in Section V.

Scheme 2: Each sensor in a hop is equally selected as the relay by the previous hop; the subcarriers and transmission power are allocated by using Algorithms 2 or 3.

Scheme 3: Each hop selects its relay sensors based on Algorithm 4; each sensor allocates subcarriers and transmission power by using MaxCh+Eq algorithm [33], where each subcarrier is assigned to the sensor with the maximal SNR and the transmission power is equally allocated among assigned subcarriers.

Scheme 4: Each sensor of a hop is equally selected as the relay by the previous hop; each sensor allocates subcarriers and transmission power by using MaxCh+Eq algorithm.

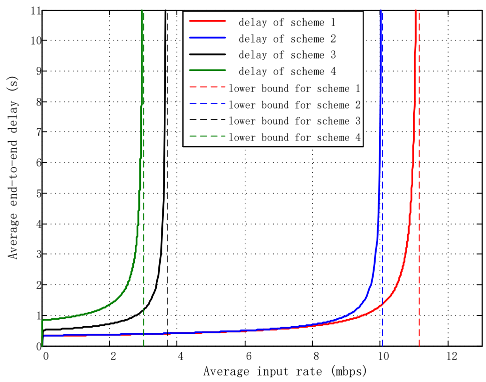

Fig. 3. Average end-to-end delay versus average input rate when $P_{m_h^{(i)}} = 50w$

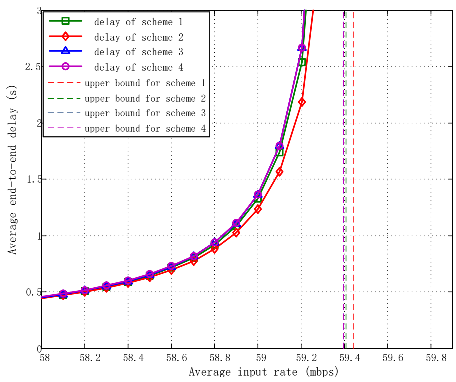

Fig. 4. Average end-to-end delay versus average input rate when $P_{m_h^{(i)}} = 200w$

Fig. 3 and **Fig. 4** compare the average end-to-end delay achieved by different transmission schemes when R varies with $P_{m_h^{(i)}} = 50w$ and $P_{m_h^{(i)}} = 200w$, respectively. The upper bound of R for each transmission scheme, calculated from Proposition 2, is marked on both figures as a dotted line, which nicely matches the simulation results. From **Fig. 3** with $P_{m_h^{(i)}} = 50w$, we observe that Scheme 1 significantly outperforms the other schemes. For example, the upper bound of R for Scheme 1 is about 11 mbps, which is much larger than that of the second best one. If the other conditions remain unchanged, both the proposed routing scheme and the resource allocation scheme can reduce the average end-to-end delay. More specifically, the proposed routing scheme can reduce the average end-to-end delay when comparing the curve of Scheme 1 to that of Scheme 2. The proposed resource allocation scheme can also reduce the average end-to-end delay when comparing the curve of Scheme 1 to that of Scheme 3. Another important observation from **Fig. 3** is that resource allocation is more effective than relay selection, when only one of the two ways is applied. This can be interpreted by the fact that the average effective goodput is very sensitive to the magnitude of received SNR, when $P_{m_h^{(i)}} = 50w$. Hence, different resource allocation schemes lead to large differences among the achieved average end-to-end delays, when $P_{m_h^{(i)}}$ is small. In **Fig. 4** with $P_{m_h^{(i)}} = 200w$, we observe that Scheme 1 performs slightly better than the other schemes. This is mainly because when $P_{m_h^{(i)}} = 200w$, increasing (decreasing) the magnitude of received SNR leads to a slow increase (decrease) of the average effective goodput. This result demonstrates that a better resource allocation scheme does not necessarily translate into a smaller average end-to-end delay when $P_{m_h^{(i)}}$ is large.

The relationship between the average end-to-end delay and $P_{m_h^{(i)}}$ is shown in **Fig. 5**, where $R = 10mbps$. The lower bound of $P_{m_h^{(i)}}$ for each transmission scheme, calculated from Proposition 2, is marked on the figure as a dotted line, which nicely matches the simulation result. The same conclusions about the comparison among all the schemes can be obtained from **Fig. 5** as those from **Fig. 3**. The average end-to-end delay of Scheme 1 decreases dramatically with $P_{m_h^{(i)}}$ increasing from 47.5 w to 70 w, because the average effective goodput is very sensitive to $P_{m_h^{(i)}}$. The average end-to-end delay of Scheme 1 decreases slowing with $P_{m_h^{(i)}}$ from 70w to 150w, because $P_{m_h^{(i)}}$ is large enough and the effect of increasing $P_{m_h^{(i)}}$ is negligible. Therefore, when $P_{m_h^{(i)}}$ is large, for example $P_{m_h^{(i)}} > 100$, increasing the value of $P_{m_h^{(i)}}$ is not an efficient way to reduce the average end-to-end delay.

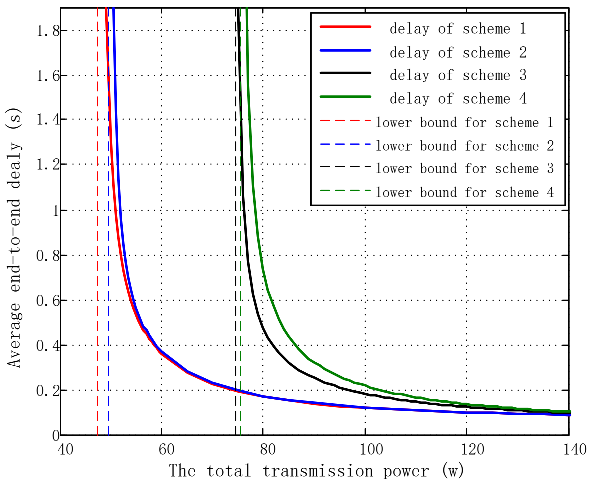

Fig. 5. Average end-to-end delay versus the transmission power when $R = 10mbps$

Fig. 6 and **Fig. 7** show the routing parameter of Scheme 1 versus R when $P_{m_h^{(i)}} = 50w$. It can be seen from both figures that the proposed optimal routing scheme tends to choose the sensor with the maximal average effective goodput as the relay when the queuing load is low. On the other hand, when the queuing load is high, the proposed optimal routing scheme tends to distribute traffic evenly among the relay sensors. The same conclusion can be also obtained from **Fig. 8** and **Fig. 9**, which investigate the relationship between the routing parameter of Scheme 1 and $P_{m_h^{(i)}}$ when $R = 10mbps$.

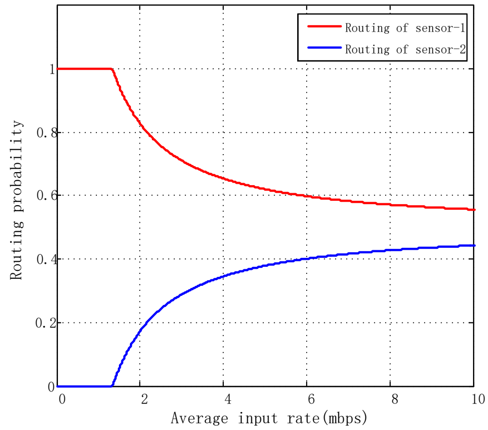

Fig. 6. Routing probability of the second hop versus average input rate when $P_{m_h^{(i)}} = 50w$

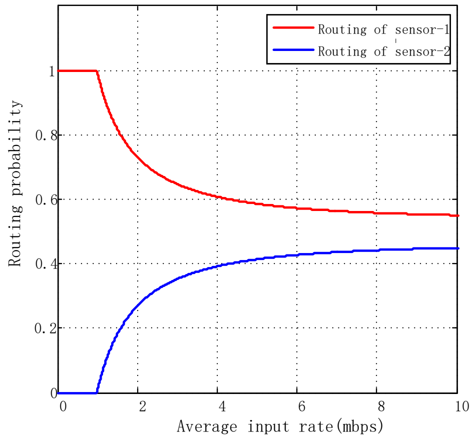

Fig. 7. Routing probability of the third hop versus average input rate when $P_{m_h^{(i)}} = 50w$

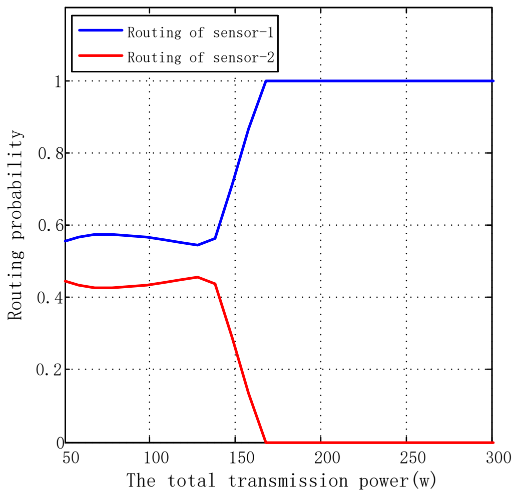

Fig. 8. Routing probability of the second hop versus the transmission power when $R = 10mbps$

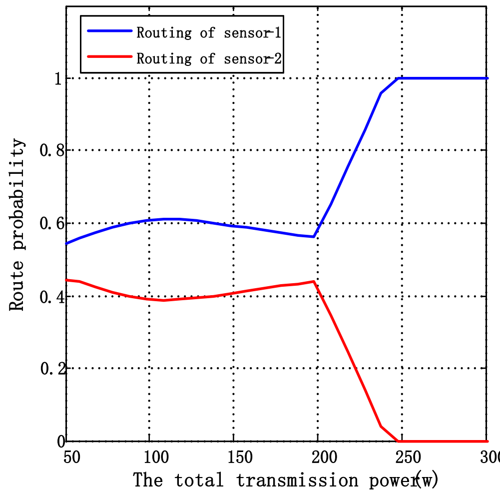

Fig. 9. Routing probability of the third hop versus the transmission power when $R = 10mbps$

7. Conclusion

In this paper, we have investigateddistributed traffic scheduling for a delay-sensitive multi-hop relay network via joint relay selection, subcarrier assignment, and power allocation. At first, we have tuned the assignment of subcarriers and transmission power for a given relay selection, and then performed relay selection for a fixed joint assignment of subcarriers and transmission power. The joint relay selection, subcarriers assignment and transmission power allocation problem for the H-th hop can also be solved by an exhaustive search over a finite set defined by the relay sensor set and available subcarrier set. Simulation results have verified that both the proposed routing scheme and the resource allocation scheme can reduce the average end-to-end delay.

Appendix

Appendix A :Proof for Proposition 1

Proposition 1 can be proved through recursion. For $P_{m_h^{(i)}} \in \left[0, \min_{f_h^{(k)} \in F'} (P'_{m_h^{(i)}, f_h^{(k)}})\right]$, $T\left(E[g_{m_h^{(i)}, f_h^{(k)}}]P_{m_h^{(i)}, f_h^{(k)}}\right)$ is an increasing and convex function in $P_{m_h^{(i)}, f_h^{(k)}}$, with $T(0) = 0$. The optimal solution for Problem 4 is assumed to be $\{P^*_{m_h^{(i)}, f_h^{(k)}}\}_{f_h^{(k)} \in F'}$. If $T\left(E[g_{m_h^{(i)}, f_h^{(2)}}](P^*_{m_h^{(i)}, f_h^{(2)}} + P^*_{m_h^{(i)}, f_h^{(2)}})\right) > T\left(E[g_{m_h^{(i)}, f_h^{(1)}}](P^*_{m_h^{(i)}, f_h^{(1)}} + P^*_{m_h^{(i)}, f_h^{(2)}})\right)$, we can obtain $T\left(E[g_{m_h^{(i)}, f_h^{(2)}}](P^*_{m_h^{(i)}, f_h^{(1)}} + P^*_{m_h^{(i)}, f_h^{(2)}})\right) > T\left(E[g_{m_h^{(i)}, f_h^{(2)}}]P^*_{m_h^{(i)}, f_h^{(2)}}\right) + T\left(E[g_{m_h^{(i)}, f_h^{(1)}}]P^*_{m_h^{(i)}, f_h^{(1)}}\right)$ based on the convexity of $T\left(E[g_{m_h^{(i)}, f_h^{(j)}}]P_{m_h^{(i)}, f_h^{(j)}}\right)$. Using the recursion method, the following conclusion is obtained that $\max_j T\left(E[g_{m_h^{(i)}, f_h^{(j)}}]P_{m_h^{(i)}}\right) > \sum_{f_h^{(k)} \in F'} T\left(E[g_{m_h^{(i)}, f_h^{(k)}}]P^*_{m_h^{(i)}, f_h^{(k)}}\right)$. Thus, the optimal solution for Problem 4 is $P^*_{m_h^{(i)}, f_h^{(k)}} = \begin{cases} P_{m_h^{(i)}} & if: f_h^{(k)} = \arg\max_{f_h^{(k)} \in F'} T\left(E[g_{m_h^{(i)}, f_h^{(j)}}]P_{m_h^{(i)}}\right) \\ 0 & else \end{cases}$. As $T\left(E[g_{m_h^{(i)}, f_h^{(j)}}]P_{m_h^{(i)}, f_h^{(j)}}\right)$ is also an increasing function in $E[g_{m_h^{(i)} f_h^{(k)}}]$, for a fixed $P_{m_h^{(i)}, f_h^{(j)}}$, then the optimal solution for Problem 4 can be equivalently simplified as $P^*_{m_h^{(i)}, f_h^{(k)}} = \begin{cases} P_{m_h^{(i)}} & if: f_h^{(k)} = \arg\max_{f_h^{(k)} \in F'} \left\{E[g_{m_h^{(i)} f_h^{(k)}}]\right\} \\ 0 & else \end{cases}$, and this completes the proof.

Appendix B :Proof for Proposition 2

By substituting (36) into (34), we can easily obtain $\sum_{i=1}^{M_h} \overline{T}_{m_h^{(i)}} > R$, and the necessity is proved.

The sufficiency can be proved in two phases:

Firstly, supposing that $\forall \overline{T}_{m_h^{(M_h)}} \le R$, then there is $n < M_h$, so that $\sum_{i=1}^{n} \overline{T}_{m_h^{(i)}} \le R$ and $\sum_{i=1}^{n+1} \overline{T}_{m_h^{(i)}} > R$. Hence, a feasible solution for Problem 6 is $\beta_{m_h^{(i)}} = \begin{cases} \overline{T}_{m_h^{(i)}} - \Delta & for\, i \le n+1 \\ 0 & for\, i > n+1 \end{cases}$, where $\Delta = \frac{\sum_{i=1}^{n+1} \overline{T}_{m_h^{(i)}} - R}{n+1}$.

Secondly, supposing that $\overline{T}_{m_h^{(1)}} > R$, then a feasible solution for Problem 6 is $\beta_{m_h^{(i)}} = \begin{cases} 1 & for\, i = 1 \\ 0 & for\, i \ne 1 \end{cases}$.

Thus, the sufficiency is proved, and this completes the proof.

Appendix C: Proof for Proposition3

Since $\forall \beta^*_{m_h^{(1)}} > 0$, it can be easily shown that $\frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}}|_{\beta_{m_h^{(i)}}=\beta^*_{m_h^{(i)}}} = \partial^*$ and $\partial^* > \overline{T}^{-1}_{m_h^{(i)}}$ based on (40). The necessity is proved.

Note that $\frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}}|_{\beta_{m_h^{(i)}}=\beta^*_{m_h^{(i)}}} > \overline{T}^{-1}_{m_h^{(M_h)}}$, for $\forall m_h^{(i)} \in \mathbf{m_h}$. As $\frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}}$ is an increasing function in $\beta_{m_h^{(i)}}$ and $\frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}}|_{\beta_{m_h^{(i)}}=0} < \overline{T}^{-1}_{m_h^{(i)}}$, $\beta^*_{m_h^{(i)}}$ must be greater than 0. Moreover, since the local optimal solutions are always globally optimal for a convex optimization problem, sufficiency is proved, and this completes the proof.

AppendixD: Proof for Proposition4

Before proceeding to prove this proposition, we rewrite objective function (33) as $\Lambda = \sum_{i=1}^{M_h} f\left(\beta_{m_h^{(i)}} \overline{T}^{-1}_{m_h^{(i)}}\right)$, where $f(x) = \frac{Rx^2}{2[1-xR]} + x$.

This proposition can be proved through contradiction. Assume there is the optimal solution with $\beta^*_{m_h^{(i)}} > \beta^*_{m_h^{(j)}}$, for $j < i$. According to (38), we can obtain $\overline{T}_{m_h^{(i)}} \le \overline{T}_{m_h^{(j)}}$. The convexity of Problem 6 ensures that $f(x)$ is an increasing and convex function in x, for all $x > 0$. Hence, under the assumption $\beta^*_{m_h^{(i)}} > \beta^*_{m_h^{(j)}}$, we can obtain $\frac{df}{dx}|_{x=\beta^*_{m_h^{(i)}}\overline{T}^{-1}_{m_h^{(i)}}} > \frac{df}{dx}|_{x=\beta^*_{m_h^{(j)}}\overline{T}^{-1}_{m_h^{(j)}}}$.

As $\overline{T}_{m_h^{(i)}} \le \overline{T}_{m_h^{(j)}}$ and $\frac{df}{dx}|_{x=\beta^*_{m_h^{(i)}}\overline{T}^{-1}_{m_h^{(i)}}} > \frac{df}{dx}|_{x=\beta^*_{m_h^{(j)}}\overline{T}^{-1}_{m_h^{(j)}}}$, it follows that there are $\Delta \in \left(0, \beta^*_{m_h^{(i)}}\right)$ such that
$$f\left[(\beta^*_{m_h^{(i)}} - \Delta)\overline{T}^{-1}_{m_h^{(i)}}\right] + f\left[(\beta^*_{m_h^{(j)}} + \Delta)\overline{T}^{-1}_{m_h^{(j)}}\right] < f\left(\beta^*_{m_h^{(i)}}\overline{T}^{-1}_{m_h^{(i)}}\right) + f\left(\beta^*_{m_h^{(j)}}\overline{T}^{-1}_{m_h^{(j)}}\right)$$
, which contradicts with the fact that $\{\beta^*_{m_h^{(1)}}, \ldots, \beta^*_{m_h^{(M_h)}}\}$ is the optimal solution. Therefore, the optimal solution should satisfy that $\beta^*_{m_h^{(i)}} \ge \beta^*_{m_h^{(j)}}$, for $i < j$, and this completes the proof.

Appendix E: Proof for Proposition5

Denote $f_i\left(\beta_{m_h^{(i)}}\right) = \frac{\beta^2_{m_h^{(i)}} R \overline{T}^{-2}_{m_h^{(i)}}}{2\left[1 - \beta_{m_h^{(i)}} R \overline{T}^{-1}_{m_h^{(i)}}\right]} + \beta_{m_h^{(i)}} \overline{T}^{-1}_{m_h^{(i)}}$. Hence, $\frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}} = \frac{df_i}{d\beta_{m_h^{(i)}}}$ and $f_i\left(\beta_{m_h^{(i)}}\right)$ is an increasing and convex function in $\beta_{m_h^{(i)}}$.

The necessity can be proved through contradiction. From Proposition 4, it can be easily shown that $\overline{T}^{-1}_{m_h^{(n)}} < \frac{\partial \Lambda}{\partial \beta_{m_h^{(i)}}}|_{\beta_{m_h^{(i)}}=\beta^*_{m_h^{(i)}}} = \frac{\partial \Lambda}{\partial \beta_{m_h^{(j)}}}|_{\beta_{m_h^{(j)}}=\beta^*_{m_h^{(j)}}} = \partial^*$, for $i, j \le n$. Assume $\frac{\partial \Lambda}{\partial \beta_{m_h^{(n)}}}|_{\beta_{m_h^{(n)}}=\beta^*_{m_h^{(n)}}} > \overline{T}^{-1}_{m_h^{(n+1)}}$. As $\frac{\partial \Lambda}{\partial \beta_{m_h^{(n+1)}}}|_{\beta_{m_h^{(n+1)}}=0} = \overline{T}^{-1}_{m_h^{(n+1)}}$ and $\frac{\partial \Lambda}{\partial \beta_{m_h^{(n)}}}|_{\beta_{m_h^{(n)}}=\beta^*_{m_h^{(n)}}} > \overline{T}^{-1}_{m_h^{(n+1)}}$, it follows that there are $\Delta \in \left(0, \beta^*_{m_h^{(n)}}\right)$ such that $f_n\left(\beta^*_{m_h^{(n)}} - \Delta\right) + f_{n+1}(\Delta) < f_n\left(\beta^*_{m_h^{(n)}}\right)$. This contradicts with the assumption that $\{\beta^*_{m_h^{(1)}}, \ldots, \beta^*_{m_h^{(M_h)}}\}$ is the optimal solution. Therefore, the necessary condition is proved.

Sufficiency

The sufficiency can also be proved via contradiction. Assume there is the optimal solution $\{\beta'_{m_h^{(1)}},\ldots,\beta'_{m_h^{(M_h)}}\}$ with $\begin{cases}\beta'_{m_h^{(i)}}>0 & for\, i\le n+1\\ \beta'_{m_h^{(i)}}=0 & for\, i>n+1\end{cases}$, and $\beta'_{m_h^{(n+1)}}=\sum_{i=1}^{n}\left(\beta^*_{m_h^{(i)}}-\beta'_{m_h^{(i)}}\right)$. Since $\beta'_{m_h^{(n+1)}}=\sum_{i=1}^{n}\left(\beta^*_{m_h^{(i)}}-\beta'_{m_h^{(i)}}\right)$, and $f_i(\beta_{m_h^{(i)}})$ is an increasing and convex function in $\beta_{m_h^{(i)}}$, we can obtain that $\sum_{i=1}^{n}\int_{\beta'_{m_h^{(i)}}}^{\beta^*_{m_h^{(i)}}}\frac{df_i}{d\beta_{m_h^{(i)}}}d\beta_{m_h^{(i)}}<\partial^*\beta'_{m_h^{(n+1)}}$. Moreover, since $\frac{df_{n+1}}{d\beta_{m_h^{(n+1)}}}|_{\beta_{m_h^{(n+1)}}=0}=\overline{T}^{-1}_{m_h^{(n+1)}}$ and $\partial^*\le\overline{T}^{-1}_{m_h^{(n+1)}}$, we can obtain that $\partial^*\beta'_{m_h^{(n+1)}}<\int_0^{\beta'_{m_h^{(n+1)}}}\frac{df_{n+1}}{d\beta_{m_h^{(n+1)}}}d\beta_{m_h^{(n+1)}}=f_{n+1}(\beta'_{m_h^{(n+1)}})$. Thus, $\sum_{i=1}^{n}\int_{\beta'_{m_h^{(i)}}}^{\beta^*_{m_h^{(i)}}}\frac{df_i}{d\beta_{m_h^{(i)}}}d\beta_{m_h^{(i)}}<f_{n+1}(\beta'_{m_h^{(n+1)}})$, and $\sum_{i=1}^{n}f_i(\beta^*_{m_h^{(i)}})<\sum_{i=1}^{n+1}f_i(\beta'_{m_h^{(i)}})$, which contradicts with the fact that $\{\beta'_{m_h^{(1)}},\ldots,\beta'_{m_h^{(M_h)}}\}$ is the optimal solution. Moreover, from Proposition 5, the same conclusion can be obtained for $\beta^*_{m_h^{(i)}}=0$, for $i\in\{n+2,\ldots,M_h\}$. The sufficiency is proved, and this completes the proof.

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YAN SHA was born in Xuzhou, P.R.China,in 1987.He received the Master degree from China Academic of Electronics and Information Technology.Now, he works in School of Medical Information and Engineering, Xuzhou Medical University. His research interests include computer networks, Ad Hoc networks, and information theory.

Junfeng Hu was born in Xuzhou, P.R.China,in 1969.He received the Ph.D. degree from China University of Mining and Technology. Now, he works in School of Medical Information and Engineering, Xuzhou Medical University. His research interests include clinical Medical Engineering and medical image processing.

Shuang Hao received the bachelor degree and master degree from Chengdu University of TCM. He currently is a lecture in Chengdu University of TCM.

Dan Wang was born in Xuzhou, P.R.China,in 1987.She received the B.S. and Ph.D. degrees from Nanjing University, in 2009 and 2014, respectively. Now, she works in School of Medical Information and Engineering, Xuzhou Medical University. Her research interests include medical informatics and data mining.